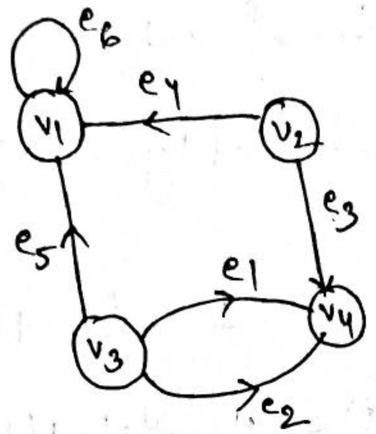
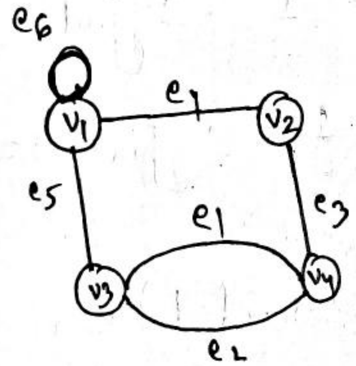
Graph-2

↳ Parallel edges: → If there are two undirected edges with same end vertices and two directed with same origin and destination, such edges are called parallel edges.

ex: →



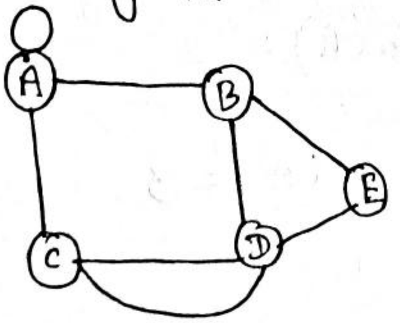
In this graph e_1 and e_2 are the two edges which are parallel as they both has same end vertices i.e v_3 and v_4.



e_1 and e_2 are the parallel edges.

↳ Undirected graph: A graph with only edges is said to be undirected graph.

ex:



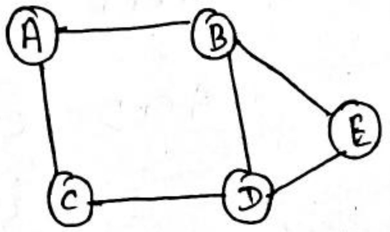
→ Undirected Graph.

↳ Directed Graph : A graph with only directed edges is said to be directed graph.

↳ Mixed graph : A graph with both undirected and directed edges is said to be mixed graph.

↳ Simple graph : A graph is said to be simple if there are no parallel and self-loop edges.

ex:

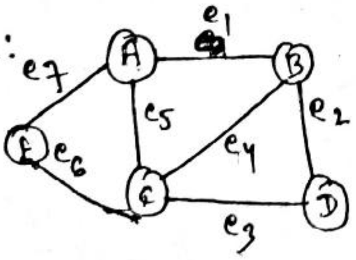


⇒ Simple graph

↳ ~~Re~~ walk : It is defined as a finite alternating sequence of vertices and edges beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it.

- No edge appears more than once in a walk
- A vertex however, appear more than once.

ex:



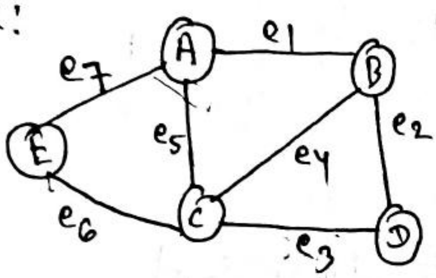
walk : A e_1 B $\overset{e_4}{\wedge}$ C e_5 A e_7 E

: A e_5 C e_4 B e_2 D e_3 C e_6 E

↳ Path : It is defined as a finite alternating sequence of vertices and edges beginning and ending with vertices, such that each edge is incident with the vertices preceding and following it.

- No edge appears more than once in a ~~walk~~ path
- No vertex ~~[illegible]~~ appears more than once in a path.

ex:



Path : $A\, e_1\, B\, e_4\, C\, e_3\, D$

: $A\, e_1\, B\, e_4\, C\, e_3\, D\, e_2\, B$ ✗
(not a path).

Graph Representations

Graph data structure is represented using following representations

↳ Adjacency Matrix

↳ Incidence Matrix

↳ Adjacency List

↳ Adjacency Matrix: In this representation, the graph is represented using a matrix of size total number of vertices by a total number of vertices. That means a graph with 4 vertices is represented using a matrix of size 4×4. In this matrix, both rows and columns represent vertices. This matrix is filled with either 1 or 0. Here, 1 represents that there is an edge from row vertex to column vertex and 0 represents that there is no edge from row vertex to column vertex.

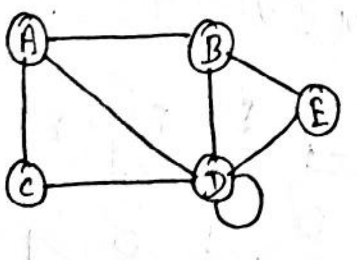
or

Suppose that $G = (V, E)$ is a graph with n vertices, then the adjacency matrix A is a $n \times n$ matrix defined as, $A = [a_{ij}]$

where

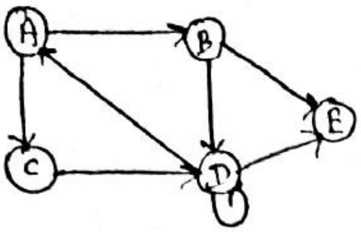
$$a_{ij} = \begin{cases} 1, & \text{if there is an edge between } v_i \text{ and } v_j. \\ 0, & \text{otherwise.} \end{cases}$$

ex: 1 Undirected graph



$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}_{5 \times 5} \end{array}$$

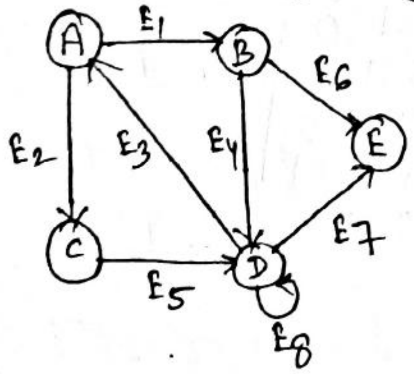
example 2: Directed graph



$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}_{5\times 5} \end{array}$$

↳ Incidence Matrix: → In this representation, the graph is represented using a matrix of size total number of vertices by a total number of edges. That means graph with 4 vertices and 6 edges is represented using a matrix of size 4X6. In this matrix, rows represent vertices and columns represent edges. This matrix is filled with 0 or 1 or -1. Here, 0 represents that the ~~row edges is~~ row vertices is not connected to column edges, 1 represents that the column edge is connected as the outgoing edge to row vertex and -1 represents that the ~~row~~ column edge is connected as the incoming edge to row vertex.

ex:



$$A = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{cccccccc} E_1 & E_2 & E_3 & E_4 & E_5 & E_6 & E_7 & E_8 \end{array} \\ \begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{bmatrix} \end{array}$$